dB at f_C. Because a passive filter has an ideal gain of zero at DC (no amplification), its gain at f_C is –3 dB.

However, the gain expression for this filter is in terms of voltage, not power. The relationship between voltage gain and decibel level is a multiplier of 20 rather than 10. Therefore, rather than looking for a gain of one-half at f_C, we must find a gain equal to the square root of one half because

$$-3\text{ dB} = 20\log_{10}\frac{1}{\sqrt{2}}$$

The filter's gain expression is complex and must be converted into a real magnitude. Conveniently, the magnitude representation looks fairly close to what we are looking for.

$$|A_{FILTER}| = \frac{1}{\sqrt{1 + (2\pi fRC)^2}}$$

By setting the denominator equal to the square root of 2, it can be observed that a gain of –3 dB is achieved when $2\pi f = 1 \div RC$. Furthermore, it can be shown that the lowpass filter's gain declines at the rate of 20 dB per radian frequency decade because, for large *f*, the constant 1 in the denominator has an insignificant affect on the magnitude of the overall expression,

$$20\log_{10}\frac{1}{2\pi fRC}$$

Rather than trying to keep all of these numbers in one's head while attempting to solve a problem, it is common to plot the transfer function of a filter on a diagram called a *Bode plot*, named after the twentieth century American engineer H. W. Bode. A Bode magnitude plot for this lowpass filter is shown in Fig. 12.19. Filters and AC circuits in general also modify the phase of signals that pass through them. More complete AC analyses plot a circuit's phase transfer function using a similar Bode phase diagram. Phase diagrams are not shown, because these topics are outside the scope of this discussion.

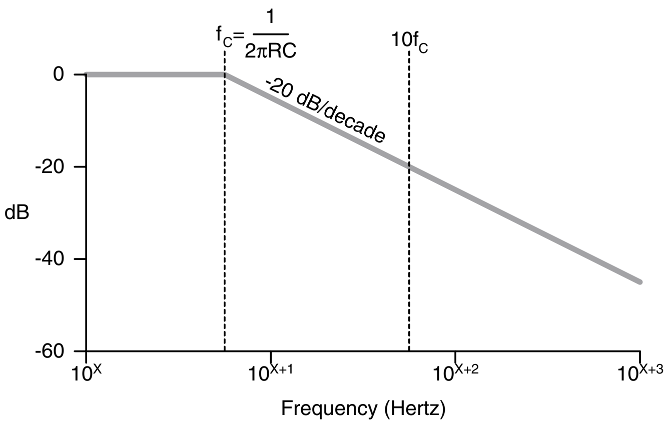

FIGURE 12.19 Bode magnitude plot for RC lowpass filter.

This Bode plot is idealized, because it shows a constant gain up to f_C and then an abrupt roll-off in filter gain. A real magnitude calculation would show a smooth curve that conforms roughly, but not exactly, to the idealized plot. For first-order evaluations of a filter's frequency response, this idealized form is usually adequate.

A filter's *passband* is the range of frequencies that are passed by a filter with little or no attenuation. Conversely, the *stopband* is the range of frequencies that are attenuated. The trick in fitting a filter to a particular application is in designing one that has a sufficiently sharp roll-off so that unwanted frequencies are attenuated to the required levels, and the desired frequencies are passed with little attenuation. If the passband and desired stopband are sufficiently far apart, a simple first-order filter will suffice, as shown in Fig. 12.20. Here, the undesired noise is almost three decades beyond the signal of interest. The RC filter's 20-dB/decade roll-off provides an attenuation of up to 60 dB, or 99.9 percent, at these frequencies.

As the passbands and stopbands get closer together, more complex filters with sharper roll-offs are necessary to sufficiently attenuate the undesired frequencies while not disturbing those of interest. Incorporating additional AC elements into the filter design can increase the slope of the gain curve beyond f_C. A second-order lowpass filter can be constructed by substituting an inductor in place of the series resistor in a standard RC circuit as shown in Fig. 12.21.

The LC filter's gain can be calculated as follows:

$$A = \frac{Z_C}{Z_C + Z_L} = \frac{\frac{1}{2\pi fC}}{\frac{1}{2\pi fC} + 2\pi fL} = \frac{1}{1 + (2\pi f)^2 LC}$$

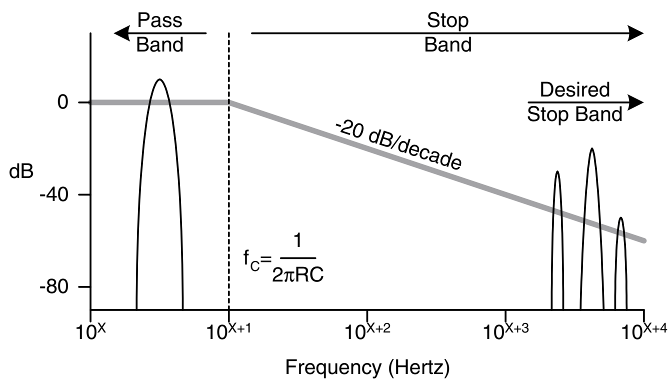

FIGURE 12.20 Widely separated pass and stopbands.

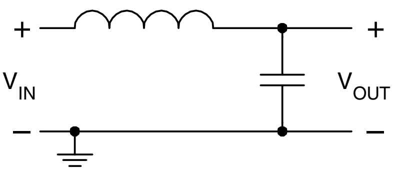

FIGURE 12.21 Second-order LC lowpass filter.